Assignment 4 Solution

Exercise 1:

Let p be you have hyperglycemia
Let q be you have hypertension
Let r be you have headaches
Let s be you have polydipsia
Let t be you feel dizzy

1. $p \vee q \rightarrow r \wedge s$ Premise
2. $p \vee q \rightarrow r$ Simplification (1)
3. $r \rightarrow t$ Premise
4. $\neg t$ Premise
5. $\neg r$ Modus Tollens (3) and (4)
6. $\neg(p \vee q)$ Modus Tollens (2) and (5)
7. $\neg p \wedge \neg q$ De Morgan's Law (6)
8. $\neg p$ Simplification (7)

Exercise 2:

a)

1. Parallelogram → 2 pairs of parallel sides Premise
2. Square → parallelogram Premise
3. Trapezoid → 1 pair of parallel sides Premise
4. Quadrilaterals → 4 sides Premise
5. **¬(1 pair of parallel sides) → ¬Trapezoid** **Contrapositive**
6. **¬(2 pair of parallel sides) → ¬Parallelogram** **Contrapositive**
7. **Trapezoid → ¬Parallelogram** **Hypothetical Syllogism (3) and (6)**
8. **Parallelogram → ¬ Trapezoid** **Hypothetical Syllogism (1) and (5)**
9. **Square → 2 pairs of parallel sides** **Hypothetical Syllogism (1) and (2)**

Conclusions: squares have 2 pairs of parallel sides. Trapezoids are not parallelograms. Parallelograms are not trapezoids. If we assume that parallelograms, squares and trapezoids are quadrilaterals, then by Hypothetical Syllogism also we can conclude that each of them have 4 sides.

b) Let W(x) be I work day x
Let S(x) be day x is sunny
Let P(x) be day x is partly sunny

1.	∀x (W(x) → S(x) ∨ P(x))	Premise
2.	W(Monday) ∨ W(Friday)	Premise
3.	¬S(Tuesday)	Premise
4.	¬P(Friday)	Premise
5.	**S(Mon) ∨ P(Mon)) ∨ S(Fri)**	**Modus Ponens (1) and (2)**
6.	**(W(Tues) ∧ P(Tues)) ∨ (¬W(Tues)**	

c) Let p be I am cheered up
Let q be I am upset
Let r be I make all people around me motivated
Let s be I make all people around me overjoyed

1.	p ∨ q	Premises
2.	¬ q	Premises
3.	**p**	**Disjunction Syllogism (1) and (2)**
4.	p → r ∨ s	Premises
5.	**r ∨ s**	**Modus Ponens (3) and (4)**

Conclusions: I make all people around me motivated, overjoyed or both

d)

1.	∀x (Healthy(x) → ¬TastesGood(x))	Premise
2.	Healthy(Tofu) → ¬TastesGood(Tofu)	Universal Instantiation (1)
3.	Healthy(Tofu)	Premise
4.	¬TastesGood(Tofu)	Modus Ponens (2) and (3)
5.	∀x (Eat(x) → TastesGood(x))	Premise
6.	¬Eat(Tofu)	Premise
7.	¬Healthy(Cheeseburgers)	Premise

Conclusions: Tofu does not taste good is concluded from (2) and (3). We can't conclude anything about cheeseburgers since we don't know if it tastes good or not, all what we know is that it is not healthy.

Exercise 3:

a) let R(x) be x is a real number

1. $\forall x ((x^2 \neq 1) \wedge R(x) \rightarrow x \neq 1)$ Premise
2. $R(a) \wedge (a^2 \neq 1) \rightarrow a = 1$? Premise
3. $\exists x ((x^2 \neq 1) \wedge R(x) \rightarrow x = 1)$ Contradiction (1) and (3)

⇨ The argument is not valid

b) Let s be you are sick
Let h be you are hungry
Let c be you are feeling cold

1. $s \wedge h \rightarrow c$ Premise
2. $c \wedge h \rightarrow s$? Premise

It is a fallacy to assume if c and h are true, we can conclude anything about s, since the c being true doesn't conclude anything…and h doesn't have any shown effect on s…

⇨ The argument is not valid

Exercise 4:

Line 3 and 5 have the same error (we can't use simplification since we have disjunction and not conjunction)

Exercise 5:

1. $\forall x (A(x) \vee B(x))$ Premise
2. $\forall x ((\neg A(x) \wedge B(x)) \rightarrow C(x))$ Premise
3. $\neg A(c) \wedge B(c)) \rightarrow C(c)$ Universal Instantiation (2)
4. $A(c) \vee \neg B(c) \vee C(c)$ Law of Implication (2)
5. $A(c) \vee B(c)$ Universal Instantiation (1)
6. $A(c) \vee C(c)$ Resolution (4) and (5)
7. $\neg C(c)$ Premise
8. $\neg C(c) \rightarrow A(c)$ Law of Implication (6)

9. A(c) Modus Ponens (7) and (8)
10. $\forall x (\neg C(x) \rightarrow A(x))$ Universal Generalization (8)

Exercise 6:

1. $q \rightarrow (u \wedge t)$ Premise
2. $u \rightarrow p$ Premise
3. $q \rightarrow u \wedge t \wedge p$ Hypothetical Syllogism (1) and (2)
4. $(p \wedge t) \rightarrow (r \vee S)$ Premise
5. $q \rightarrow (r \vee S)$ Hypothetical Syllogism (3) and (4)
6. $\neg S$ Premise
7. $q \rightarrow r$ Disjunctive Syllogism (5) and (6)

Exercise 7:

a) Samir is a student in this class. Premises
Samir is from Russia. Premises
Everyone from Russia has had a flu at least once. Premises
Someone in this class has had a flu. Existential Generalization

Let S(x) be x is a student of this class
Let R(x) be x is from Russia
Let F(x) be x has had a flu

1. S(Samir) Premise
2. R(Samir) Premise
3. $\forall x (R(x) \rightarrow F(x))$ Premise
4. $R(Samir) \rightarrow F(Samir)$ Universal Instantiation (3)
5. F(Samir) Modus Ponens (2) and (4)
6. $S(Samir) \wedge F(Samir)$ Conjunction (1) and (5)
7. $\exists x (S(x) \wedge F(x))$ Existential Generalization (6)

b) Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken CMPS 200.
Every student who has taken CMPS 200 can take CMPS 212.

All five roommates can take CMPS 212

Let CMPS200(x) be x has taken CMPS 200
Let CMPS212(x) be x can take CMPS 212

1.	CMPS200(Melissa) ∧ CMPS200(Aaron) ∧ CMPS200(Ralph) ∧ CMPS200(Veneesha) ∧ CMPS200(Keeshawn)	Premises
2.	∀x (CMPS200(x) → CMPS212(x))	Premises
3.	CMPS200(Melissa) → CMPS212(Melissa)	Universal Instantiation (2)
4.	CMPS200(Aaron) → CMPS212(Aaron)	Universal Instantiation (2)
5.	CMPS200(Ralph) → CMPS212(Ralph)	Universal Instantiation (2)
6.	CMPS200(Veneesha) → CMPS212(Veneesha)	Universal Instantiation (2)
7.	CMPS200(Keeshawn) → CMPS212(Keeshawn)	Universal Instantiation (2)
8.	CMPS200(Melissa)	Simplification (1)
9.	CMPS200(Aaron)	Simplification (1)
10.	CMPS200(Ralph)	Simplification (1)
11.	CMPS200(Veneesha)	Simplification (1)
12.	CMPS200(Keeshawn)	Simplification (1)
13.	CMPS212(Melissa)	Modus Ponens (3) and (8)
14.	CMPS212(Aaron)	Modus Ponens (4) and (9)
15.	CMPS212(Ralph)	Modus Ponens (5) and (10)
16.	CMPS212(Veneesha)	Modus Ponens (6) and (11)
17.	CMPS212(Keeshawn)	Modus Ponens (7) and (12)
18.	CMPS212(Melissa) ∧ CMPS212(Aaron) ∧ CMPS212(Ralph) ∧ CMPS212(Veneesha) ∧ CMPS212(Keeshawn)	Conjunction (13), (14), (15), (16), (17)

c) All self-centered people suffer from the illusion of control. Premises
Natasha, our classmate is self-centered. Premises
Some of our classmates suffer from the illusion of control. Existential Generalization

Let S(x) be x is self-centered
Let C(x) be x is in our class
Let I(x) be x suffers from the illusion of control

1.	∀x (S(x) → I(x))	Premises

2. S(Natasha) → I(Natasha) Universal Instantiation (1)
3. S(Natasha) Premises
4. C(Natasha) Premises
5. I(Natasha) Modus Ponens (2) and (3)
6. C(Natasha) ∧ I(Natasha) Conjunction (4) and (5)
7. ∃x (C(x) ∧ I(x)) Existential Generalization (6)

d) All Math classes taught by professor John are wonderful. Premises
Professor John is giving a course this semester. Premises
The Math department has wonderful courses this semester. Modus Ponens

Let J(x) be John is giving course x
Let W(x) be course x is wonderful
The domain of x is all math courses

1. ∀x (J(x) → W(x)) Premises
2. ∃x (J(x)) Premises
3. J(c) → W(c) Universal Instantiation (1)
4. J(c) Universal Instantiation (2)
5. W(c) Modus Ponens (3) and (4)
6. ∃x (W(x)) Existential Generalization (5)

Exercise 8:

The problem in concluding B(George, George) given "there exists an s such that B(s, George)", is in the *there exists* itself, since this means that its is true from some *s,* and not for any value of *s*, so replacing *s* by George is fallacious.

Exercise 9:

a) Correct
   1. ∀x (EnrolledInUni(x) → Dorm(x)) Premises
   2. EnrolledInUni(Mia) → Dorm(Mia) Universal Instantiation (1)
   3. ¬Dorm(Mia) Premises
   4. **¬EnrolledInUni(Mia)** **Modus Tollens**

b) Not Correct
   1. TouchScreen → Fragile Premises

2. ¬TouchScreen — Premises

We can't conclude anything about the laptop since the hypothesis is false (the conclusion can have any truth value)

c) Not Correct

1. Android → Java — Premises
2. Java — Premises

We can't conclude anything about the phone since the conclusion is true (the hypothesis can have any truth value)

d) Correct

1. Plant → ProduceGlucose — Premises
2. Palm → Plants — Premises
3. **Palm → ProduceGlucose** — **Hypothetical Syllogism**

Exercise 10:

1. $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ — Premises
2. $(p \vee q)$ — Simplification (1)
3. $(\neg p \vee q)$ — Simplification (1)
4. $(p \vee \neg q)$ — Simplification (1)
5. $(\neg p \vee \neg q)$ — Simplification (1)
6. q — Resolution (2) and (3)
7. ¬q — Resolution (4) and (5)
8. $q \wedge \neg q$ — Conjunction (6) and (7)
9. False (not satisfiable) — Negation Law (8)